UNIVERSITY OF CONNECTICUT

SCHOOL OF ENGINEERING

STORRS, CONNECTICUT

ADAPTIVE CONTROL OF MULTIVARIABLE SYSTEMS

D. P. Lindorff

Department of Electrical Engineering

September 1969

This work has been sponsored by the
National Aeronautics and Space Administration
Research Grant NGL 07-002-002

ADAPTIVE CONTROL OF MULTIVARIABLE SYSTEMS

D. P. Lindorff
Department of Electrical Engineering
University of Connecticut
Storrs, Conn. 06268

I INTRODUCTION

In this paper, Park's adaptive design [1] is applied to multivariable systems. Although the method proposed by Parks has been developed in general form by Winsor and Roy [2], unresolved questions were raised which lead to the conclusion that the method has limitations, particularly if an attempt is made to apply the design to the multivariable class of problems. These limitations are encountered if, in causing the plant to track a model, the adaptive signals act only at the plant inputs, rather than directly on the plant parameters. Since it is not usually possible to alter the plant parameters directly, these limitations are considered to be of practical importance.

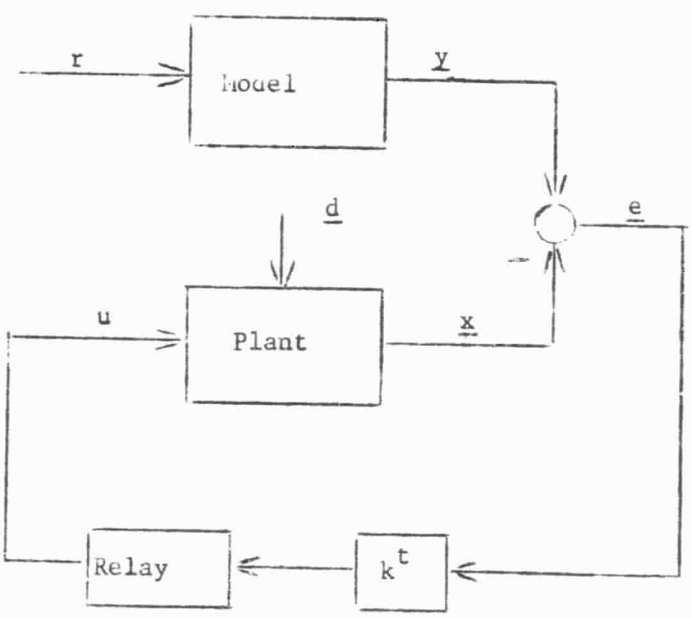
It is shown, for the linear time-invariant system having the same number of inputs as outputs, that a stable adaptive control system can be designed if the state variables are related to the outputs as phase variables, and the number of outputs is no greater than the number of inputs. It is further shown that instability can result if a certain relationship is not preserved between the control inputs of the plant. If the model is noninteracting, a simplification in the design can be achieved which depends upon the use of a partitioned Liapunov function as used in [3].

By Parks' method, an imperfectly identified plant is caused to track a model with guaranteed stability. However, the effect of a disturbance upon the adaptive system has not been previously considered. It is shown that the method in [3] can be used in some cases as a modification of the Parks' design so that asymptotic stability can be assured in the presence of disturbance.

II FORMULATION OF INPUT MODIFICATION - SINGLE VARIABLE PLANT

It is convenient at the outset to differentiate between two methods of design based on the application of Liapunov's direct method. For this purpose Phillipson has proposed using the terms <u>input modification</u> and <u>feedback synthesis</u> [4]. In both of these approaches, the technique involves the selection of an appropriate Liapunov function, and the generation of a control law which assures that the time derivative of this function will be negative, at least outside of some region enclosing the system's equilibrium. By such means it has been shown that a plant can be caused to track a model with bounded (perhaps zero) error in spite of inexact specification of plant parameters and, in the case of input modification, a bounded disturbance [1,3,5,6].

Since use is to be made of both these methods, the two design approaches will be summarized in this and the following section. At the outset the discussion will be limited to the single variable plant. Consider first <u>input modification</u>.



Input Adaptive Design

Figure 1

As shown in Figure 1, the design will be developed in terms of a relay controller.

The model and plant are represented respectively by the linear equations

$$\underline{\dot{y}} = A_m \underline{y} + \underline{b}_m r \tag{2.1}$$

and

$$\underline{\dot{x}} = A_p \underline{x} + \underline{b}_p u + \underline{d} \tag{2.2}$$

for which $\underline{x} = [x_i]$, $\underline{y} = [y_i]$, $\underline{b}_m = [b_{mi}]$, $\underline{b}_p = [b_{pi}]$, $\underline{d} = [d_i]$ are n vectors, r and u are scalar inputs, and $\underline{d}$ is a disturbance. If the tracking error is defined by $\underline{e} = \underline{y} - \underline{x}$, then the differential equation describing motion in error space becomes

$$\underline{\dot{e}} = A_m \underline{e} + \underline{f} \tag{2.3}$$

wherein

$$\underline{f} = A\underline{x} + \underline{b}_m r - \underline{b}_p u - \underline{d}$$

and

$$A = A_m - A_p.$$

If it is assumed that coefficients of A_p are known to be within certain bounds, then it follows for $A = [a_{ij}]$ that bounds on each a_{ij} are also known. It is also assumed that bounds on each b_{pi} are given, as well as its sign.

In order to design for stability of (2.3), it is convenient to select as a candidate for a Liapunov function the quadratic form

$$V = \underline{e}^t P \underline{e}, \tag{2.4}$$

where it is assumed that $P = [p_{ij}]$ is a real symmetric positive-definite matrix. The time derivative of V becomes, with the use of (2.3),

$$\dot{V} = -\underline{e}^t \, Q \, \underline{e} + 2\underline{e}^t \, P \, \underline{f} \tag{2.5}$$

where

$$-Q = A_m^t \, P + P \, A_m. \tag{2.6}$$

Now, by a theorem of Liapunov [7], if A_m is a stability matrix, i.e., the model is assumed to stable, then for any positive definite symmetric Q, there is a positive definite symmetric P which satisfies (2.6). Hence if $\underline{e}^t P \underline{f} \leq 0$, then $\dot{V}$ is negative definite, and (2.3) has a stable equilibrium.

In order to see how u may be used to control the sign of $\underline{e}^t P \underline{f}$, it is convenient to write

$$\underline{e}^t P \underline{f} = \underline{\gamma}^t \underline{f}, \tag{2.7}$$

where $\underline{\gamma} = [\gamma_i]$, $\underline{f} = [f_i]$. In explicit form it is seen that

$$\underline{e}^t P \underline{f} = \sum_{i=1}^{n} \gamma_i f_i \tag{2.8}$$

where

$$\gamma_i = \sum_{j=1}^{n} p_{ji} e_j \tag{2.9}$$

and

$$f_i = \sum_{j=1}^{n} a_{ij} x_j + b_{mi} r + b_{pi} u - d_i . \tag{2.10}$$

As a first attempt at controlling the sign of $\underline{e}^t P \underline{f}$, we shall group terms containing u together, so that (2.7) becomes

$$\underline{e}^t P \underline{f} = \left(\sum_{i=1}^{n} \gamma_i b_{pi}\right) u + \underline{g}\ (\underline{x}, r, \underline{d}, \underline{y}). \tag{2.11}$$

Here $\underline{g}$ contains the conglomerate of terms which do not contain u. Now if the magnitude of the term containing u is made large enough to override the magnitude of $\underline{g}$, it would be possible to cause the sign of $\underline{e}^t P \underline{f}$ to be negative by making the sign of u equal to the sign of $-\Sigma\, \gamma_i b_{pi}$. However, the sign of the term multiplying u cannot be determined for all values of $\underline{e}$ if elements b_{pi} are not known exactly. Hence, even though the term containing u may be large enough, its sign cannot be controlled completely, due to parameter uncertainty.

Another possibility is to attempt to control the sign of each term, $\gamma_i f_i$, in (2.8). According to (2.10) this would require for i = 1, ...,n that

$$|u| \geq \left| \frac{\Sigma a_{ij} x_j + b_{mi} r - d_i}{b_{pi}} \right|$$

$$\operatorname{sgn} b_{pi} u = - \operatorname{sgn} \gamma_i \tag{2.12}$$

Since u is a scalar, the sign requirements in (2.12) can be met only if for each $f_i \neq 0$, the sign of each associated γ_i has the same sign at every instant of time. This is ruled out since the P matrix in (2.4) would then have to be semi - definite. The conclusion is that $\underline{f}$ can have but one element which is not identically zero. Hence if P is positive definite as assumed, the state variables $\underline{x}$ and $\underline{y}$ must be phase variables.* Although a semidefinite P matrix has been used to generate a control law for input-modification systems having non-phase variable structure [8], it will be seen that the semidefinite form cannot be used to accommodate feedback synthesis. For this reason the positive-definite P will be assumed throughout.

It will be shown below that phase-variable form is required if feedback synthesis is applied to the single variable plant whose parameter values cannot

---

* The meaning here is that $\underline{x}$ must be obtained by taking derivatives of the output unless the plant is structured in phase variable form, in which case the states can be obtained from direct measurements.

be manipulated.

III FORMULATION OF FEEDBACK SYNTHESIS - SINGLE VARIABLE PLANT

The feedback synthesis approach to adaptive control is developed around the same equations of the model and the plant as appear in (2.1) and (2.2), subject to the assumption that A_p, $\underline{b}_p$ are fixed but that parameter values are not known exactly. Since the method by itself is not concerned with disturbance rejection, it is assumed for the present that $\underline{d} = \underline{0}$. The tracking error is again defined as in (2.3), the objective being to cause the equilibrium at $\underline{e} = \underline{0}$ to be asymptotically stable.

In the feedback synthesis approach originally formulated by Parks, the automatic adjustment of plant parameters by direct manipulation was included in the design. The development which follows is specialized in that direct manipulation is not permitted for reasons of practical importance. Hence, in the following development all adaptive signals must be applied to the plant through the control variable, u. It will be seen that, for each plant parameter which has a value different from that of a corresponding parameter in the model, an adaptive signal is generated so as ultimately to cause the plant to follow the model with zero error. This is accomplished by introducing as a candidate for a Liapunov function, the expression

$$V = \underline{e}^t P \underline{e} + \sum_{j=1}^{n+1} \phi_j^2 \lambda_j^{-1} \tag{3.1}$$

where each term, $\phi_j^2 \lambda_j^{-1}$, represents a scalar function to be defined appropriately so that V is positive definite if P is positive definite.* The time derivative of V now becomes

$$\dot{V} = -\underline{e}^t Q \underline{e} + 2\underline{e}^t P\underline{f} + 2 \sum_{j=1}^{n+1} \frac{\phi_j \dot{\phi}_j}{\lambda_j} . \tag{3.2}$$

wherein $\underline{f}$ and Q are defined as in (2.3) and (2.6) respectively.

The design approach is to define each ϕ_j so that

$$\underline{e}^t P\underline{f} + \sum_{j=1}^{n+1} \frac{\phi_j \dot{\phi}_j}{\lambda_j} = 0. \tag{3.3}$$

It follows that $\dot{V}$ will then be negative definite if Q is chosen to be negative definite as before, and V is a Liapunov function if P is a solution to (2.6), with A_m defined as a stability matrix.

To find ϕ_i, attention is directed to a term $\gamma_i f_i$ in (2.8), rewritten here for convenience (with $d_i = 0$);

$$\gamma_i f_i = \gamma_i \left[\sum_{j=1}^{n} a_{ij} x_j + b_{mi} r + b_{pi} u \right] . \tag{3.4}$$

---

* n+1 terms are required so that an adaptive signal can be generated from each state variable in $\underline{x}$ and the input r. Each λ_i is a positive constant.

If u is expressed in terms of n + 1 components,

$$u = \sum_{j=1}^{n+1} u_j , \tag{3.5}$$

then (3.4) can be rewritten in the form

$$\begin{aligned} \gamma_i f_i = \sum_{j=1}^{n} \ \gamma_i \ (a_{ij} \ x_j + b_{pi} \ u_j) \\ + \gamma_i \ (b_{mi} \ r + b_{pi} \ u_{n+1}) \quad . \end{aligned} \tag{3.6}$$

Now let the components of u in (3.5) be defined as

$$u_j = k_j \ x_j, \ j=1 \ldots n,$$

and

$$u_{n+1} = k_r \ r, \text{ with } k_{j+1} = k_r.$$

Then (3.6) takes the form

$$\begin{aligned} \gamma_i f_i = \sum_{j=1}^{n} \gamma_i \ (a_{ij} + b_{pi} \ k_j) \ x_j \\ + \gamma_i \ (b_{mi} + b_{pi} \ k_r) \ r. \end{aligned} \tag{3.7}$$

If ϕ_j is defined according to

$$\begin{aligned} \phi_j &= (a_{ij} + b_{pi} \ k_j), \quad j=1, \ldots, n, \\ \phi_{n+1} &= b_{mi} + b_{pi} \ k_r, \end{aligned} \tag{3.8}$$

where the terms k_j, k_r are permitted to be time dependent, then time derivatives of (3.8) become

$$\dot{\phi}_j = b_{pi} \ \dot{k}_j, \ j=1, \ldots, n, \tag{3.9}$$

$$\dot{\phi}_{n+1} = b_{pi} \ \dot{k}_r \ .$$

If (3.7) and (3.9) are substituted into (3.3), and for the moment $\underline{f}$ is assumed to have only one component, f_n, which is not identically zero, then (3.3) becomes

$$\sum_{j=1}^{n} \gamma_n (a_{nj} + b_{pn} k_j) x_j + \gamma_n (b_{mn} + b_{pn} k_r) r$$

$$+ \sum_{j=1}^{n} (a_{nj} + b_{pn} k_n) b_{pn} \dot{k}_j \lambda_j^{-1} \tag{3.10}$$

$$+ (b_{mn} + b_{pn} k_r) b_{pn} \dot{k}_r \lambda_{n+1}^{-1} = 0.$$

It is readily seen that (3.10) can be satisfied if

$$\dot{k}_j = - \frac{\lambda_j}{b_{pn}} x_j \gamma_n, \quad j=1, \ldots, n$$

and (3.11)

$$\dot{k}_r = - \frac{\lambda_{n+1}}{b_{pn}} r \gamma_n.$$

An embodiment of these equations in an adaptive control system is shown in Figure 2 where the notation is adopted that

$$\underline{k}_x = [k_j], \quad j=1, \ldots, n$$

$$H = \begin{bmatrix} h_1 & & & 0 \\ & h_2 & & \\ & & \ddots & \\ 0 & & & h_n \end{bmatrix}$$

where

$$h_j = - \frac{\lambda_j}{b_{pn}}, \quad j=1, \ldots, n,$$

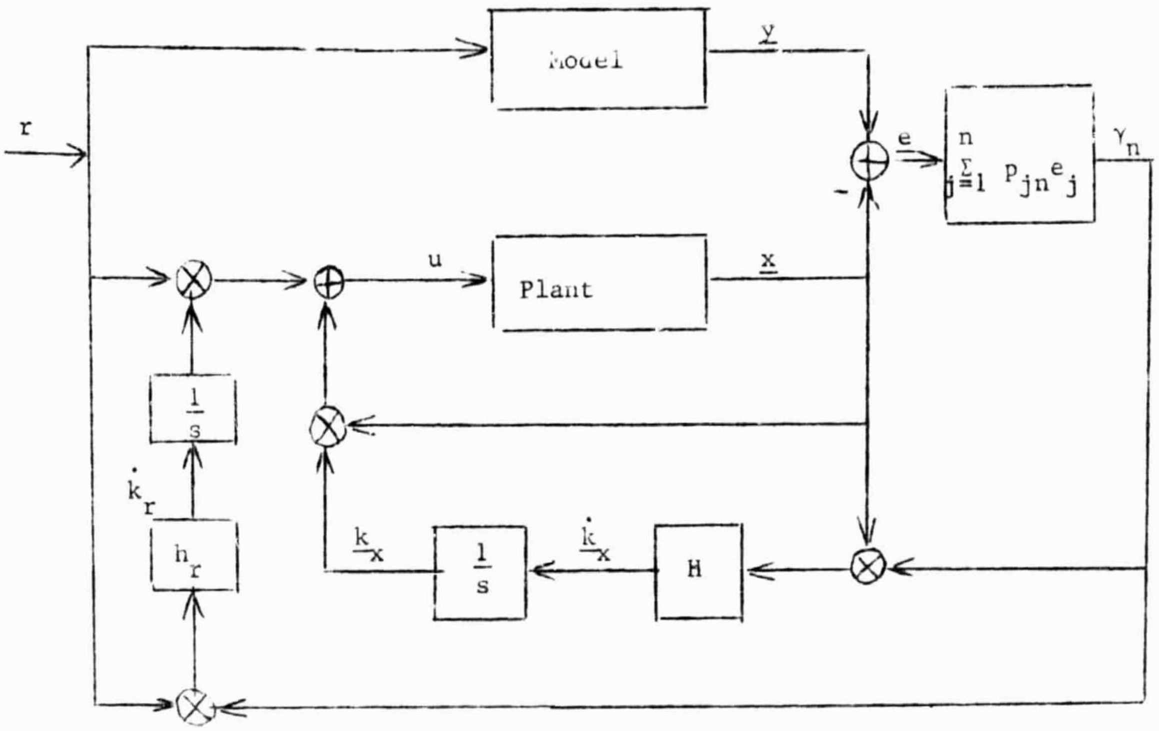
and for j=n+1

$$h_{j+1} = h_r = - \frac{\lambda_{n+1}}{b_{pn}}$$

Although the term b_{pn} is not known exactly, its sign must be known. Then the magnitude of each h_j can be specified arbitrarily, since each λ_j can have any desired positive constant value.

Since asymptolic stability is assured relative to $\underline{e}$, it follows that the gains will ultimately attain values which cause the dynamics from r to $\underline{x}$ to be idential to that from r to $\underline{y}$.

Finally the assertion is made that it is in fact necessary to assume that all but one component of $\underline{f}$ is identically zero. If there were a second term in $\underline{f}$



Feedback Synthesis Applied
to Single Variable Plant

Figure 2

other than zero say f_q, for which $b_{pq} \neq 0$, then according to (3.4) there would be a need to generate u so as to nullify the term $\gamma_q f_q$ as well. But $\gamma_q \neq \gamma_n$ if P is positive definite. Hence the scalar u cannot be generated to serve both purposes. Clearly the terms in $\gamma_q f_q$ cannot be cancelled if $b_{pq} = 0$.
<u>The conclusion is reached that a single variable plant must be in phase variable form in order that feedback synthesis can be applied through the action of the (scalar) control variable.</u>

To remove this restriction requires that P be semidefinite as in [8]. In this case the gains in the adaptive loops would acquire a set of values which would restrict $\underline{e}$ to motion on a hyperplane; however, there would be no guarantee that $\underline{e}$ would eventually reach the origins as is desired.

IV FEEDBACK SYNTHESIS APPLIED TO MULTIVARIABLE SYSTEMS

In this section the more general problem is considered in which the model and plant equations have the respective forms

$$\dot{\underline{y}} = A_m \underline{y} + B_m \underline{r}$$
$$\underline{z} = C_m \underline{y} \qquad (4.1)$$

and

$$\dot{\underline{x}} = A_p \underline{x} + B_p \underline{u}$$
$$\underline{w} = C_p \underline{x} \qquad (4.2)$$

where $\underline{z}$, $\underline{w}$ are vectors representing the model and plant outputs respectively. The newly introduced matrices are $B_m = [b_{mij}]_{n,p}$, $B_p = [b_{bij}]_{n,p}$, $C_p = [c_{pij}]_{m,n}$, $C_m = [c_{mij}]_{m,n}$, $\underline{r} = [r_i]_{p,1}$, $\underline{u} = [u_i]_{p,1}$, $\underline{w} = [w_i]_{m,1}$, $z = [z_i]_{m,1}$. Subscripts outside the brackets signify matrix dimensions.

The tracking error is again defined as $\underline{e} = \underline{y} - \underline{x}$, so that

$$\dot{\underline{e}} = A_m \underline{e} + \underline{f} \qquad (4.3)$$

with

$$\underline{f} = A \underline{x} + B_m \underline{r} - B_p \underline{u} \qquad (4.4)$$

and

$$A = A_m - A_p. \qquad (4.5)$$

For this case (3.1) must be generalized so that the candidate for a Liapunov function is now

$$V = \underline{e}^t P \underline{e} + \sum_{i=1}^{p} \sum_{j=1}^{q} \phi_{ij}^2 \lambda_{ij}^{-1} \tag{4.6}$$

where a set of ϕ's has been provided for each element of $\underline{u}$, and the value of q depends upon the specific problem. It follows that

$$\dot{V} = \underline{e}^t Q \underline{e} + 2\underline{e}^t P \underline{f} + 2 \sum_{i=1}^{p} \sum_{j=1}^{q} \phi_{ij} \dot{\phi}_{ij} \lambda_{ij}^{-1} \tag{4.7}$$

where, as in (2.7),

$$\underline{e}^t P \underline{f} = \underline{\gamma}^t \underline{f} \tag{4.8}$$

$$= \sum_{i=1}^{n} \gamma_i f_i .$$

In this case

$$f_i = \sum_{j=1}^{n} a_{ij} x_j + \sum_{j=1}^{p} b_{mij} r + \sum_{j=1}^{p} b_{pij} u_j , \; i=1, \ldots, n. \tag{4.9}$$

By extending the same argument which was used in Section II to show that all but one component of $\underline{f}$ must be identically zero for single variable plants, it can be seen, in the multivariable case and with P positive definite, that n-p elements of $\underline{f}$ must be identically zero. This is because there are only p elements of $\underline{u}$, and therefore there can be only p non-zero terms, $\gamma_i \; f_i$, in (4.8) if $\underline{u}$ is to be used successfully in nullifying $\underline{e}^t P \underline{f}$ in $\dot{V}$.

The application of feedback synthesis to the multivariable system proceeds along the same lines as for the single variable case, the difference being that each of the p terms, $\gamma_i f_i$, which are not identically zero must be nullified by a different element of $\underline{u}$. The problems which are peculiar to the multivariable system will now be discussed.

First it is to be shown that the number of outputs must not be more than the number of inputs, i.e. $m \leq p$. To this end, it is noted that, if the plant is to follow the model with zero error, it is necessary that $C_p = C_m$ in (4.1), (4.2). Otherwise $\underline{w} \neq \underline{z}$ even though $\underline{e} = \underline{0}$. Since it is assumed that parameters of the plant are not known exactly, it cannot in general be assumed that C_p is known. If, however, the state variables are derived as derivates of the respective outputs, then C_p and C_m become identity matrices. It follows that, for each output and its associated phase variables, there will be one element of $\underline{f}$ which is not identically zero. But the number of components of $\underline{u}$ must be at least as great as the number of elements of $\underline{f}$ which are not identically zero. In the sequel we shall assume that the number of inputs are equal to the number of outputs, i.e. $m = p$.

A problem which, unless it is recognized, may cause serious difficulty, concerns the possible existence of more than one element of $\underline{u}$ in expressions for f_i. To introduce this problem, consider the case in which $\underline{u}$ is composed of two terms, , u_1 and u_2. Then based on the foregoing discussion, there will be only two elements of $\underline{f}$ which are not identically zero. We can therefore write

$$\underline{e}^t P \underline{f} = \gamma_i f_i + \gamma_j f_j \qquad (4.10)$$

where

$$f_i = g_i + b_{p11} u_1 + b_{p12}\, u_2,$$

$$f_j = g_j + b_{p21}\, u_1 + b_{p22}\, u_2.$$

Following the method outlined in Section III, it is appropriate to express u_1 and u_2 in terms of additive components which can be associated with various adaptive gains. Let u_1 and u_2 be written as

$$u_i = u_{i1} + u_{i2}, \; i = 1,\, 2. \qquad (4.11)$$

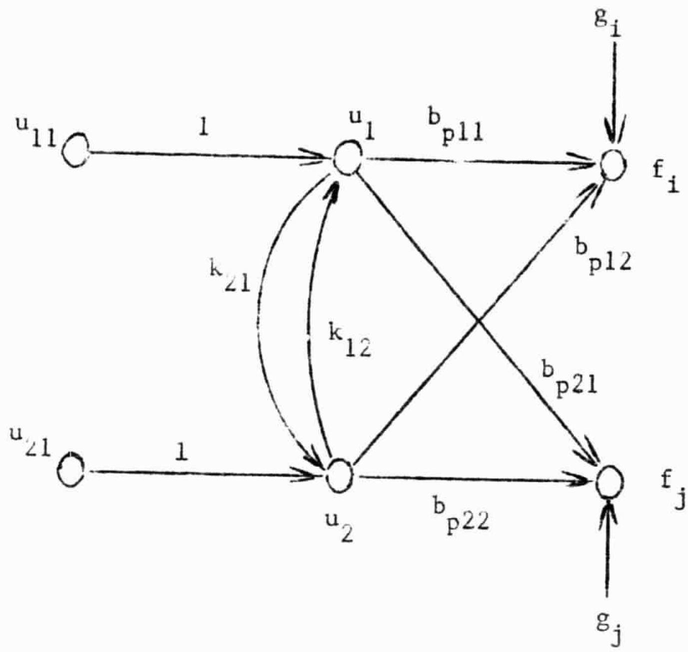
If it is assumed that u_{11} and u_{22} contain the components which are designed to nullify g_i and g_j, then u_{12} and u_{21} can be used to nullify the terms $b_{p12}\, u_2$ and $b_{p21}\, u_1$ respectively. The scheme is diagrammatically represented in Figure 3, for the simple case of two control variables. The method is valid provided that the feedback loop composed of $k_{21} k_{12}$ does not cause instability.

For the general case in which the components of the control variables are

$$u_i = \sum_{i=1}^{p} u_{ij}, \; j=1, \ldots, n ,$$

and the associated adaptive gain terms are treated as constants, the stability is governed by the roots of the characteristic equation

$$\begin{vmatrix} -1 & k_{12} & k_{13} & \cdots & k_{1p} \\ k_{21} & -1 & k_{23} & \cdots & k_{2p} \\ k_{31} & k_{32} & -1 & \cdots & k_{3p} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ k_{p1} & k_{p2} & k_{p3} & \cdots & -1 \end{vmatrix} = 0 \qquad (4.12)$$



$$u_{12} = k_{12}\ u_2$$

$$u_{21} = k_{21}\ u_1$$

Interaction of Two Control Inputs

Figure 3

There is of course in the real system some dynamics associated with each of the gain terms k_{ij}. Since these gains will in general be time variable, the stability analysis is greatly complicated. However, if the gains are slowly varying relative to the small time constants associated with them, it is reasonable to use as a stability criterion the requirement that the time varying roots of (4.12) have negative real parts.

It should be noted that the stability problem is avoided if the plant is constructed so that the B_p matrix in (4.2) has the triangular form.

$$B_p = \begin{bmatrix} b_{11} & & & \\ b_{21} & b_{22} & 0 & \\ \cdot & & \cdot & \\ \cdot & & \cdot & \\ \cdot & & \cdot & \\ b_{p1} & b_{p2} & \cdots & b_{pp} \end{bmatrix} \qquad (4.13)$$

In this case all the feedback paths are broken since the determinant in (4.12) has the same form as (4.13), i.e. $k_{ij} = 0$ if $b_{ij} = 0$.

V USE OF PARTITIONED LIAPUNOV FUNCTION FOR NONINTERACTING MODEL

It has been shown in [3] that, in the case of multivariable systems, simplification in the form of (2.6) can be made if the model is noninteracting. More specifically, if A_m can be partitioned in the diagonal form

$$A_m = \begin{bmatrix} A_{11} & & & & 0 \\ & \ddots & & & \\ & & A_{ii} & & \\ & & & \ddots & \\ 0 & & & & A_{pp} \end{bmatrix} \qquad (5.1)$$

and $Q = [Q_{ii}]$ is similarly partitioned, then P as a solution to (2.6) will have a corresponding partitioned diagonal form, so that

$$P = \begin{bmatrix} P_{11} & & & & & & 0 \\ & \cdot & & & & & \\ & & \cdot & & & & \\ & & & P_{ii} & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ 0 & & & & & & P_{pp} \end{bmatrix} . \tag{5.2}$$

By this means, V in (4.6) can be expressed as

$$V = \sum_{i=1}^{p} V_i \tag{5.3}$$

where

$$V_i = \underline{e}_i^t \; P_{ii} \, \underline{e}_i + \sum_{j=1}^{q} \phi_{ij}^2 \; \lambda_{ij}^{-1} \tag{5.4}$$

and $\underline{e}_i$ is a subvector of $\underline{e}$ with dimension which is conformable with P_{ii}. Then

$$\dot{V}_i = - \underline{e}_i^t \, Q_{ii} \, \underline{e}_i + 2\underline{e}_i^t \, P_{ii} \, \underline{f}_i + 2 \sum_{j=1}^{q} \frac{\phi_{ij} \, \dot{\phi}_{ij}}{\lambda_{ij}} \tag{5.5}$$

in which $\underline{f}_i$ is a subvector of $\underline{f}$ with the same dimension as $\underline{e}_i$. Based on the results of Section IV it is known that each subvector, $\underline{e}_i$, must be in phase variable form. Therefore only one element of each $\underline{f}_i$ is not identically zero, as is required for implementation of the control law. Because of the diagonal partitioned form of P it follows that each P_{ii} is positive definite if P is positive definite. Hence it is possible to achieve asymptotic stability by requiring that each term $\dot{V}_i$, i=1,...,p, be negative definite. This offers a considerable simplification in the derivation of the control law.

VI DISTURBANCE REJECTION

As stated earlier, the method of feedback synthesis does not in itself provide a means of controlling against disturbance inputs. This is because the method requires that a measurement of the variable in question be available. It will be observed, however, that this is not the case in input modification. As can be seen from (2.12), the requirement on u is simply that it be greater than the largest value assumed by the right-hand side. Hence, if d_n is the term of interest, as would be the case with phase-variable form, a component of u, say u_{n+2}, must be employed which satisfies the relationships

$$\left| u_{n+2} \right| \geq \max \left| \frac{d_n}{b_{pn}} \right| \tag{6.1}$$

$$\operatorname{sgn} b_{pn} u_{n+2} = - \operatorname{sgn} \gamma_n$$

Hence only bounds on d_n must be know.

By adding the tern u_{n+2} to the expression for u in (3.5), the control variable becomes

$$u = \sum_{j=1}^{n+2} u_j \tag{6.2}$$

where u_{n+2} is the output of a relay which switches on the sign of γ_n. It follows that the effect of d_n can be eliminated provided that the output level of the relay satisfies the relationship $L \geq \max \left| d_n / b_{pn} \right|$.

The method has application to the multivariable problem only if it is possible to satisfy certain inequalities between the relay outputs associated with the various control variables as noted in [3]. A straightforward application of this design modification can be made to the case in which the B_p matrix of the plant has the form of (4.13). Otherwise the presence of feedback paths with nonlinear (relay) elements presents a stability problem which is considered to be beyond the scope of this paper.

VII CONCLUSIONS

In extending the application of Parks' adaptive control system design to multivariable systems certain limitations were imposed. It is shown, for the practical case in which direct manipulation of plant parameters is not allowed, that the state variables must be related to the outputs as phase variables, and the number of outputs must be no more than the number of inputs. A critical problem concerns the possibility of instability arising from the existence of feedback loops arising from the adaptive gains operating on the control variables. A solution to this problem is found for a class of multivariable systems. However, more work must be done to gain confidence in the method when applied to the broader class of problems. Finally it is shown that by a modification of the design, it is possible in some cases to control against disturbances. In particular, the ideas contained in this paper apply without exception to the single variable plant, this being a special case of the multivariable case.

Although no simulation studies are reported here, initial results indicate that further work is required in order that a reasonable design can be achieved. For example, although stability may be assured, it is not always a simple matter to select the adaptive gains and the P matrix so that reasonable convergence time is assured. This is a problem for further study.

REFERENCES

1. Parks, P. C., Lyapunov Redesign of Model Reference Adaptive Control Systems, Trans. IEEE, AC-11, V11, N3 (1966)

2. Winsor, C. A., and Roy, R. J., Design of Model Reference Adaptive Control Systems by Liapunov's Second Method (corresp); Trans. IEEE, AC 13, N2, 204 (1968)

3. Lindorff, D. P., Control of Nonlinear Multivariable Control Systems, Trans, IEEE, AC-12, N 11, 506-515 (1967)

4. Phillipson, P. H., Design Methods for Model-Reference Adaptive Systems, Proc. Inst. of Mechanical Engrs. , V 183, pt 1 (1969)

5. Monopoli, R. V., Synthesis Technique Employing the Direct Method, Trans. IEEE, AC-10, N 3, 369-370 (1965)

6. Taylor, T. M., Determination of a Realistic Error Bound for a Class of Imperfect Controllers, JACC Preprints, Ann Arbor, Mich. 522-538 (1968)

7. Hahn, W., Theory and Applications of Liapunov's Direct Method, Prentice-Hall, Englewood Cliffs, N. J., (1963)

8. Lindorff, D. P., Relay Control of Systems with Parameter Uncertainties and Disturbances, Automica, Pergomon Press, Vol. 5, November (1969)